Close *Friday*: 14.7
Close Tues: 15.1, 15.2
Close next Thur: 15.3

Global Max/Min

Consider a surface z = f(x,y) over a particular region R on the xy-plane.

An **absolute/global maximum** over R is the largest z-value over R.
An **absolute/global minimum** over R is the smallest z-value over R.

Key fact (Extreme value theorem)
The absolute max/min occur at either

1. A critical point, or
2. A boundary point.

Example: Let R be the triangular region in the xy-plane with corners at (0,-1), (0,1), and (2,-1). Above this triangular region, find the absolute max and min of

$$f(x,y) = \frac{1}{4}x + \frac{1}{2}y^2 - xy + 1$$

Entry Task

Do Step 1: Find the critical points

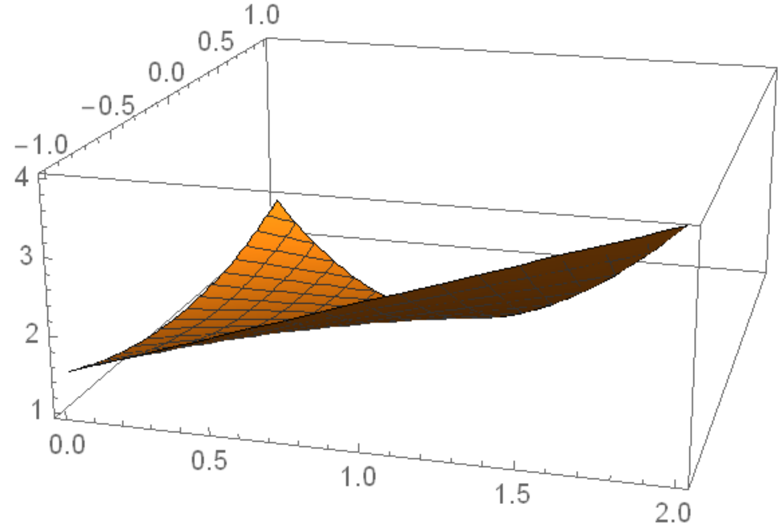
How to find the absolute max/min

Step 1: Find critical points inside region.

Step 2: Find critical numbers and corners above each boundary.

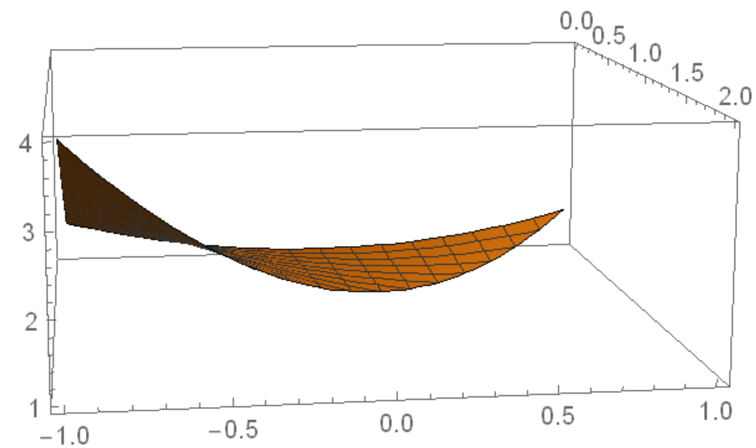
i) For each boundary, give an equation in terms of x and y. Find intersection with surface.

ii) Find critical numbers and endpoints for this one variable function. Label “corners”.



Step 3: Evaluate the function at all points you found in steps 1 and 2.

Biggest output = global max
Smallest output = global min



Example:

Find the absolute max/min of

$$f(x,y) = x^3 - 12x + y^2$$

over the region

$$x \geq 0, x^2 + y^2 \leq 9.$$

Homework hints

In applied optimization problems,

(a) Identify what you are optimizing!
(b) Label Everything.
(c) Identify given facts (constraints)
(d) Use the constraints and labels to give a 2 variable function for the objective.

HW Examples:

1. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to (4,2,0).

Objective: Minimize **distance** from (x,y,z) points on the cone to the point (4,2,0) given that $z^2 = x^2 + y^2$.

2. Find the dimensions of the box with volume 1000 cm^3 that has minimum surface area.

Objective: Minimize **surface area** given that volume is 1000.

3. You want to build aquariums with slate for the base and glass for the sides (and no top).
   Assume slate costs \$5 per in^2 and glass costs \$1 per in^2.
   If the volume must be 1000 in^3, then what dimensions will minimize cost?

Objective: Minimize **cost** when volume needs to be 1000.